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THE ASSESSMENT OF LIQUID PROPELLANT INJECTORS

Part I. Atomisation: its measurements and influence on
Combustion Efficiency of Rocket Motors

G. K. Adams

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E.R.D.E. TECHNICAL REPORT 28/R/48.

The Assessment of Liquid Propellant Injectors

Part I. Atomisation: its measurements and
influence on combustion efficiency
of rocket motors.

by

G.K. Adams.

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LIST OF SYMBOLS.

Unless stated otherwise C.G.S. units are used
in this report.

SECTION I.

- d = diameter of droplet
 \bar{d} = mean diameter of spray
 N_i = number of droplets diameter d_i
 $V_{\text{and}} S$ = total volume and surface of N droplets
 R = volume fraction of spray having diameter $> d$
 d_0 = size constant in Rosin Rammler equation
 n = distribution constant in Rosin Rammler equation
 \dot{V}_b = volume rate of burning of propellant
 k = proportionality constant
 $S_v, N_v \text{ etc.}$ = surface, number etc., per unit volume of liquid
 \dot{V} = volume rate of input of propellant
 v = volume fraction
 β = index in the equation relating rate of burning to diameter of drop
 t = time (seconds)
 M = rate of change or flow of mass
 \dot{m} = " " " " " " " per unit area.
 λ = thermal conductivity
 C_p = specific heat at constant pressure
 Θ = non-dimensional temperature defined in text
 T = temperature in $^{\circ}\text{A}$
 L = heat of vaporisation
 D = diffusion constant
 ν = kinematic viscosity
 v = flow velocity

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SECTION II.

ρ = density
 γ = surface tension
 l, L = representative lengths
 μ = viscosity
 Q = volume flow rate
 t_o = thickness of film at orifice
 P = pressure in atmospheres
 λ = wavelength
 r = radius

SECTION III.

I = illumination
 B = brightness
 $N.A$ = numerical aperture
 M = magnification
 d = depth of field
 Z = diameter of circle of confusion
 λ = wavelength
 T = transmission

APPENDIX A. As in Section I.

APPENDIX B. As in Section I plus :

w = rate of reaction per unit volume per second
 α = mass fraction of reactant A etc.
 B = frequency factor in first order rate equation
 E = energy of activation.

INTRODUCTION.

One of the advantages of the rocket motor over other propulsion units lies in its ability to achieve a high rate of conversion of chemical energy into kinetic energy in a relatively small volume. In order to maintain the high rate and density of combustion required, the propellant is injected into the combustion chamber in the form of a spray consisting of a large number of small droplets. This spray has a large surface area per unit volume which facilitates the transfer of heat from the combustion gases to the liquid propellant. Thus the high rate of evaporation from the liquid to the gaseous phase, necessary to maintain combustion, is achieved.

In the case of multi-component propellants mixed in the combustion chamber, the dispersion of each component is a step towards their mutual interaction.

It is desirable to have methods of assessing the atomising properties of injectors for liquid propellants which can be correlated with measurements of the efficiency of the injector in the combustion system for which it is designed. The methods which have been used in other fields for the measurement of drop size in sprays are reviewed in this report, but first it is desirable to examine the influence of droplet size on the combustion of propellant sprays, because the selection of a method suitable for the assessment of rocket injectors will be governed by the form of the relation between drop size and rate of combustion. Sprays produced by methods employed in combustion engineering contain droplets of which the size may vary over a considerable range and it is necessary to characterise both the mean droplet size and the degree of deviation from the mean. The definition of these quantities will depend upon their relation to the mechanism of combustion.

The calculation of the overall rate of combustion of the propellant spray from possible relations between the rate of combustion of an individual droplet and the dimension of that droplet is discussed in the first section of this report. In order to do this a greatly simplified picture of the propellant spray and the process of combustion is necessary. It is assumed that the process of atomisation is instantaneous, that is, the propellant is injected in the form of discrete droplets, the initial size and distribution of which can be assumed, whereas in reality the break up of the injected propellant is occurring in the combustion chamber simultaneously with its combustion. It is to be expected that the physical factors entering into the combustion of mono-propellants should be much simpler than in bi-propellant systems. Initially, therefore, discussion has been confined to the combustion of mono-propellants. The combustion of systems in which fuel and oxidant are injected separately is of such complexity that it merits a separate discussion which, it is hoped, may form a subsequent part of this dissertation. In the case of mono-propellants it is assumed that each droplet burns independently of the rest of the spray, and therefore the overall rate is given by the sum of the combustion of all droplets in the combustion chamber. It is then possible to calculate the rate of burning per drop as a function of the thermal properties of the propellant and the size of the drop. The assumptions needed to give an algebraic solution are too drastic to permit of

/quantitative

quantitative calculation of the rate of combustion of any particular propellant system but should give an indication of the dependance of combustion upon droplet diameter and therefore permit the examination of the effect of changes in the mean droplet size and inhomogeneity of the propellant spray.

The process of atomisation depends not only on the injector but also on the conditions existing in the combustion chamber and on certain properties of the liquid propellant. Since it will be more convenient to test the atomising properties of injectors in the absence of combustion and with liquids other than the propellant for which the injector has been designed, it is desirable to discuss the effect of these changes on the degree of atomisation, and therefore the relation of the latter to the conditions of operation. In the second section the literature on the atomisation of liquids is reviewed with particular reference to the effect of liquid properties such as viscosity and surface tension and to the operating conditions such as injection pressure, air density, etc.

The third section reviews methods of measurement of drop size in liquid sprays and discusses their applicability to rocket injectors. It is desirable that the method or methods adopted should be simple and speedy and of general application to the varied types of spray which will be tried in the early stages of rocket development. It is unlikely, however, that there will be a requirement for a purely routine method of checking the atomisation of production types for control purposes in the same way as might be required for, say, flow rate. This is because small differences in the degree of atomisation will have a negligible effect on the overall efficiency of the rocket motor. The need is more for a method of determining large differences resulting from a change in type or scale of the injection system in development work, and for research into the importance of atomisation and injection methods in rocket combustion. There is relatively little information in the literature on this subject, and what there is lacks detail, suggesting the need for further research in this field.

SECTION I.

INFLUENCE OF DROPLET SIZE ON COMBUSTION.

In the last two decades a number of papers (1, 2, 3, 4) have been published dealing with the influence of the degree of subdivision of fuel sprays in compression ignition engines. It has generally been taken for granted that the rate of combustion of the fuel spray is directly related to the total surface of the spray, and for purposes of comparison between different sprays a mean diameter is defined such that the actual spray is replaced by a fictitious homogeneous spray, having the same ratio of total volume to total surface. This mean diameter was first used by the German engineer Sauter (1); and, to distinguish it from other mean diameters it is often termed the Sauter mean diameter; this is defined as follows :

$$\bar{d}_s = \frac{\sum_{i=1}^N n_i d_i^3}{\sum_{i=1}^N n_i d_i^2} = \frac{6V}{S} \quad (1)$$

Where n_i is the number of droplets having a diameter d_i , and the summations are taken over the total number of droplets N ; V and S are the total volume and surface respectively. It is obvious that an infinite number of sprays have the same Sauter mean diameter, differing in the range of dispersion of the drop sizes about this mean, and attempts were made to get up some number characteristic of the lack of uniformity of the spray.

These early attempts (2) usually involved a number derived from the standard deviation of the droplet distribution, no methods having been developed for measuring the distribution of droplet size in fuel sprays with any precision. In recent years, however, in connection with the development of spray nozzles for gas turbines (5) and oil furnaces (6), it has been observed experimentally that the distributions of droplet sizes in sprays from swirl type spray nozzles fit the Rosin-Rammler type of distribution law, first applied to pulverised coal dust (7) which has the form

$$R = e^{-\left(\frac{d}{d_0}\right)^n} \quad (2)$$

Where R is the cumulative volume fraction with diameter greater than d , and d_0 and n are the size and distribution constants respectively; the diameter d_0 is that above which $1/e^n$ of the total spray volume lies; the value of n defines the 'spread' of the distribution and is found experimentally to take values greater than two, the larger values of n indicating more uniform distribution. Experimentally the relation has been applied to sieve fractionation of solidified fuel droplets where relatively large samples of the spray can be examined and the volume distribution is determined directly. For values of n greater than two, most of the spray volume lies in the range $0.1 < \frac{d}{d_0} < 2.5$ as can be seen from the table given on page 35 of

/Appendix A

Appendix A to this report. It is shown there that, if the cumulative volume distribution follows equation (2) then the probability distribution functions for other parameters of the spray such as surface, diameter and number per unit volume, can be simply expressed in terms of tabulated functions, subject to certain restrictions upon the values of the distribution constant n .

While it is necessary to remember that an injector operating under combustion chamber conditions will form sprays of markedly different mean droplet size and distribution from those found in 'cold' tests, it is plausible to assume that the form of the distribution law will be similar. Then if the dependance of the rate of burning of the propellant drops upon their size is assumed, the effect of changes in the size distribution and distribution of the spray can be examined. For instance, if the rate of burning of the spray, referred to unit volume of propellant, is proportional to the surface area per unit volume

$$\dot{V}_b = k S_v = \frac{6k}{\bar{d}_s}$$

where \bar{d}_s is the Sauter mean diameter as defined in equation (1) and k is a constant, the rate of burning therefore increases with decreasing mean diameter. It can be shown (P.37) that for a spray obeying the Rosin Rammler distribution law

$$\bar{d}_s = d_g / \Gamma(1 - \frac{1}{n}) \quad (3)$$

where $\Gamma(1 - \frac{1}{n})$ is the Gamma function defined as

$$\Gamma(1+x) = \int_0^{\infty} t^x e^{-t} dt, \quad x > -1$$

or as $\Gamma(1+x) = (x)!$

values of this function are tabulated (42)

As $n \rightarrow \infty$, $\Gamma(1 - \frac{1}{n}) \rightarrow 1$

and $n \rightarrow 1$, $\Gamma(1 - \frac{1}{n}) \rightarrow \infty$

As n varies between ∞ and 2, $\Gamma(1 - \frac{1}{n})$ increases from 1 to approximately 2; thus the rate of burning of the spray increases with decreasing homogeneity. This refers to the initial rate of burning of the propellant spray and this as R.P. Probert (8) has remarked is of less importance than the steady state of combustion

/in

in a chamber into which propellant is continuously injected.

If the time spent by the propellant in the combustion chamber is unrestricted in the steady state a constant volume of propellant will be burning with a volume rate of burning equal to the volume rate of input. The droplet size distribution of the propellant and its combustion properties will determine the volume of propellant in the combustion chamber. The ratio of this volume to the combustion chamber volume represents the density of loading of the combustion system; it is plausible to assume that this, for a given propellant and combustion chamber and injector design, should be held constant.

It is shown in Appendix A to this report that if the rate of burning normal to the surface of the propellant droplets is independent of their diameter, so that the overall rate of combustion is proportional to the surface area, then the variation of diameter of any droplet with time can be expressed as

$$d = d_i - kt \quad (4)$$

and the volume of propellant in the combustion chamber in the steady state is

$$V = \frac{\dot{V}d_0}{4k} \cdot \sqrt[1+\frac{1}{n}} \quad (5)$$

assuming that the state of the propellant droplet distribution at the moment of injection can be represented by (2)

The variation of this volume with throughput \dot{V} and burning constant k is to be expected. It is however, remarkable since $\sqrt[1+\frac{1}{n}}$ is almost independent of n for values greater than twom that it should depend mainly on the size constant d_0 and be but slightly affected by the spread of the distribution.

However, in any practical case the time that can be spent by any droplet in the combustion chamber is limited and droplets over a certain initial size may be ejected from the combustion chamber before they have burnt completely. It can be shown that the fraction of the propellant spray injected at time $t = 0$ remaining unconsumed at a time $t > 0$ can be represented as

$$v_t = \int \left(\frac{k_{gt}}{d_0^{3-\beta}} \right) \quad (6)$$

assuming that variation of drop diameter with time can be represented by

$$d^{3-\beta} = d_i^{3-\beta} - k_{gt}t$$

The form of the function depends upon the values of n and β and cannot in general be given explicitly. Values of v_t for particular values of $(k_{gt})/d_0^{3-\beta}$ and n are best found from

/equation

equation (28) (Appendix A) by numerical integration. Values are tabulated below for a range of values of n . It can be seen that the fraction remaining increases with decreasing n .

Volume fraction of injected propellant unburnt at time t after injection as a function of n at $k_{\beta}t/d_0^{3-\beta} = 1$

		n		
		2	3	4
β	1	0.10	0.069	0.050
	2	0.011	0.044	0.0219

This value of $k_{\beta}t/d_0^{3-\beta}$ corresponds to complete combustion of the homogeneous spray of droplet size d_0 . The figures quoted indicate no more than the relatively small change in efficiency to be expected from changes in distribution or "spread" of the droplet sizes and the expected conclusion:

That for maximum efficiency with a limited time spent in the combustion chamber the spray distribution should be as homogeneous as possible.

Similar conclusions have been reached by R.P. Probert for the injection of oil fuels into gas turbine combustion chambers. The methods employed in Appendix A for the evaluation of mean diameters etc., are generalisations of his methods for propellant systems where the rate of combustion of drops can be related to their diameter by indicial equations of the general form

$$\dot{M} = kd^{\beta}$$

Probert assumes that the combustion in gas turbine systems is dependant upon the rate of evaporation of the fuel which is assumed to be governed by an equation similar to the psychrometric equation in its dependance on linear dimension of the drops, i.e. the rate of evaporation per drop is directly proportional to the diameter.

Since it can be argued that combustion of propellants in general is preceded by evaporation it is worth while examining the applicability of this assumption to propellant systems.

The assumption that the rate of combustion of a drop of propellant is proportional to the first power of the drop radius implies that the rate of combustion per unit surface is inversely proportional to the drop radius. Applied to mono-propellant systems this would seem to contradict experience, since in internal ballistics, the rate per unit surface is usually assumed to be independent of the geometrical configuration of the surface, and the overall rate of combustion to be proportional to the total surface area. This apparent contradiction can be resolved if we examine the process of combustion in relation to the linear scale of the combustible drop. It will be shown

/that

It will be shown that the two possible relations for the rate of burning of a drop of monopropellant $\beta = 1$ or 2

are limiting forms of the relation between rate of burning and linear dimension.. If one imagines the liquid drop to be surrounded by two zones then at the surface of the drop evaporation occurs, in the first zone the vapour is heated to its ignition temperature, and in the second, reaction occurs with the liberation of heat, part of which is conducted back to the surface of the liquid to maintain the evaporation. For convenience, it is assumed that a steady state has been reached in which the heat conducted back is just sufficient to evaporate the weight of liquid consumed in the reaction, so that the temperature of the surface of the drop is constant. It is probable that such a steady state is realised, since if the reaction zone is displaced towards the liquid surface, the heat conducted back will increase, the velocity of efflux of vapour will increase and the reaction zone will move away from the surface towards its initial position.

It is assumed that this constant surface temperature is equal to the boiling point of the liquid under the combustion chamber pressure and that the temperature distribution in the interior of the liquid is constant. The thermal diffusivity of liquids is of the order of 10^{-4} cms²/sec so that the surface temperature will fall rapidly towards the centre of the drop and it will be convenient to include the heat required to raise the liquid from its initial temperature to the surface temperature in the heat of vaporisation. The problem is then to solve the partial differential equations of heat and mass flow in the zone outside the liquid drop with certain boundary conditions. Since we are interested in the variation of rate of burning with droplet diameter rather than in calculating the absolute rate of burning, and since the form of the equations renders an exact solution laborious, the problem has been simplified by the assumption of a temperature below which the reaction is zero. This temperature is termed the ignition temperature, although it has no relation to the temperature of ignition as usually measured. Above the ignition temperature it is implicitly assumed that the variation of reaction rate with position can be approximated by a simple function of the distance from the surface, although this function is eliminated from the final result by referring the rate of combustion of the drop to that of a plane surface. If we then make the further assumptions that the ignition temperature and the rate of reaction at that temperature are independent of the radius of curvature of the propellant surface, and that parameters such as thermal conductivity and heat capacity can be assigned a mean value independent of temperature, the following relation can be derived (see Appendix B)

$$2\dot{m} = \dot{m}_0 + 2 \cdot \frac{2\lambda \ln \theta}{C_p d} + \sqrt{(\dot{m}_0 + 2 \cdot \frac{2\lambda \ln \theta_i}{C_p d})^2 - 4 \left(\frac{2\lambda \ln \theta_i}{C_p d} \right)^2} \quad (7)$$

when \dot{m} and \dot{m}_0 are the mass rates of burning per unit surface for a drop of diameter d and a plane surface respectively.

/T₀

and
$$\Theta_i = \frac{C_p(T_i - T_0 + \frac{L}{C_p})}{L}$$

T_0 being the surface temperature °A

T_i the ignition temperature °A

L the heat of evaporation per unit mass

C_p the mean constant pressure specific heat of the propellant vapour (between T_0 and T_i) per unit mass

λ is the mean thermal conductivity of the vapour between T_0 and T_i

Putting $\frac{2\lambda \ln \Theta_i}{C_p} = \varphi$ it can be seen that if

$$d \gg \frac{4\varphi}{\dot{m}_0} \quad \dot{m} \approx \dot{m}_0 + \frac{\varphi}{d} \approx \dot{m}_0 \quad (7a)$$

and if

$$d \ll \frac{4\varphi}{\dot{m}_0} \quad \dot{m} \approx \frac{\dot{m}_0}{2} + \frac{\varphi}{d} + \left(\frac{\varphi}{d}\right)^{1/2} \approx \frac{\varphi}{d}$$

It can be shown that $\frac{\varphi}{2\dot{m}_0}$ is the width of the zone in which the propellant vapour is heated to the temperature T_i in the burning of a plane surface. The rate of burning of the propellant drop is thus dependent on the surface area or on the diameter according to relative order of magnitude of the size of the drop and the width of the heating zone for the case of a plane surface.

The value of $\frac{\varphi}{\dot{m}_0}$ will depend upon the properties of the particular mono-propellant considered; it will obviously increase with decreasing rate of burning. The latter quantity can in theory be measured experimentally but the values for actual mono-propellants are not known with any accuracy. For the purpose of illustration it is sufficient to assign nominal values to the rate of burning, and the physical constants contained in φ . If we take methyl nitrate as an example it is considered that the following values are plausible at a pressure of twenty atmospheres.

$$\dot{m}_0 = 1 \text{ gram/cms}^2 \text{ sec.} \quad \lambda = 10^{-4} \text{ cals/cms}^2 \text{ sec.}^\circ \text{ grad.}$$

$$L = 100 \text{ cals/gram.} \quad T_0 = 350^\circ\text{A}, \quad T_i = 850^\circ\text{A.}$$

$$C_p = 0.5 \text{ cals/gram}^\circ. \quad \Theta_i = 3.5 \text{ and } \frac{\varphi}{\dot{m}_0} \approx 5\mu$$

There is a certain amount of experimental evidence for the value of \dot{m}_0 , the value of T_i however is very speculative. There is indirect evidence that the bulk of the reaction in combustion systems occurs at temperatures close to the final temperature, Semenov (9), Wolfhard and Klaukens (10). Boys and Corner (11) have calculated the temperature and extent of reaction as a function of the distance from the propellant surface for the burning of colloidal propellants; it would appear that a negligible fraction of the rate determining reaction takes place below 850°A . Fortunately φ is relatively insensitive to changes in the value

/of

of T_1 . The values of λ , C_p , etc., are probable by analogy rather than by experiment, and the computed value of \dot{Q}/\dot{m}_0 should be taken as an order of magnitude rather than a numerical value. The inference is that the assumption that the rate of burning is directly proportional to the surface area is true only for propellant drops above about 200μ in the case of methyl nitrate. It is thus open to question whether this law can be applied to the spray distribution existing in rocket combustion chambers, particularly since although representative injectors are found to give sprays under atmospheric conditions in which the bulk of drops be between 100 and 400μ , the distribution under combustion chamber conditions is unknown. It is probable that the increase in temperature of the liquid will decrease the mean diameter and the spread of the distribution by decreasing the liquid surface tension and viscosity, but the change due to the alteration in gas flow conditions cannot be estimated. If it is assumed that the bulk of the propellant will have drop diameters in a range between 10 and 200μ neither of the approximate forms (7a) of the burning law will apply. The form of equation (7) makes the calculation of the burning characteristics of the distribution by the methods outlined in Appendix B and referred to in the first part of this section somewhat tedious. It can be shown however, by a log/log plot of \dot{m} against d that the rate of burning per drop may be expressed with fair accuracy within a limited range of diameter by a relation of the form

$$\dot{M} = k d^{\beta} \quad (8)$$

where β has a value between one and two

Thus for $\dot{Q}/\dot{m}_0 \approx 5\mu$

$$\dot{M} = k_1 d^{1.4}, \quad 1 < d < 10\mu$$

$$\text{and } \dot{M} = k_2 d^{1.8}, \quad 10 < d < 200\mu$$

By the use of these approximations the relations deduced in Appendix B for the relevant mean diameter, volume of propellant in combustion chamber, volume fraction unconsumed after a certain time, etc., can be extended to droplet size ranges in which the limiting forms of the combustion equation are inadequate.

So far we have not considered the possible effect of any relative motion between the propellant and the combustion chamber gases. In order to achieve a state of atomisation the propellant is given an initial velocity of the order of some tens of metres per second; as it proceeds along the chamber the propellant spray is decelerated by resistance forces, and the combustion chamber gases are accelerated both by the shared momentum of the spray and by the conversion of chemical energy into thermal and translational energy. Although the relation between the resistance forces and the drop dimensions and relative velocity are known for conditions under which no combustion takes place, it will probably be considerably changed for the case of a burning drop, since the burnt gases may have a velocity normal to the surface of similar order to that of the mean relative velocity between the drop and the combustion chamber gases.

/However

However, it can be concluded that any relative velocity will have been reduced to a low order for the greater part of the combustion (it will be noted that the decelerating force is inversely proportional to some power of the drop diameter; therefore this conclusion becomes more likely the smaller the mean drop size of the spray) and the residual relative velocity will have only a small effect upon the combustion of a mono-propellant spray.

This last conclusion is not, however, likely for bi-component propellant systems, where fuel and oxidant are injected separately, and the rate of combustion will depend upon the rate of mixing of the two components. Then the flow conditions in the combustion chamber become of great importance. Since these are not amenable to calculation, a detailed analysis of the combustion is not practicable, and such conclusions as can be drawn are general, and only approximate. The rate of combustion at any point will be proportional to the product of the concentrations of the reactants at that point, and a stable zone of combustion can be formed only where a balanced and constant rate of supply of each component is possible. The latter will depend upon the initial evaporation and subsequent diffusion and convection of each component from the liquid drops to the combustion zone, and it is pertinent to examine how these processes depend upon the diameters of the individual droplets.

Since the equations determining heat flow and mass flow are similar in form, the requirement for a simultaneous solution of these equations will not affect the order of the dependence upon the dimensions of the system, and only one need be considered. The rate of mass flow across a spherical surface concentric with the drop, within which the rate of reaction is inappreciable, will be directly proportional to the radius of the drop only if the distance of the surface from the drop is large compared to the drop radius; thus the linear dependence of evaporation upon drop size assumed by Probert will hold only if reaction occurs at a distance from the drop large compared to the drop radius; as the reaction zone distance approaches the drop diameter, the rate of evaporation will become proportional to the surface area of the drop.

Under conditions in which the drop has a relative translational motion with respect to the gaseous medium, the rate of evaporation is increased by a term which is dependent upon the Reynolds number of the flow conditions.

From measurements of the rate of evaporation from spheres in a flowing medium by Frozling (12), Kramers (13) has obtained the following relationship between the dimensionless numbers representing the rate of evaporation, the flow conditions and the properties of the gaseous medium:

$$\frac{m d}{D} = 2 + 0.55 (Pr)^{\frac{1}{3}} (Re)^{\frac{1}{2}} \quad (9)$$

for $2 < Re < 800$ and $0.6 < Pr < 2.7$

where m is the rate of evaporation per unit surface per unit concentration difference and d the drop diameter, D the diffusion coefficient

coefficient. The Prandl number Pr , representing the properties of the gaseous medium, is defined as : $Pr = \frac{\nu C_p}{\lambda}$

λ being the thermal conductivity and ν the kinematic viscosity, and the Reynolds number representing the condition of flow as

$$Re = \frac{v d}{\nu}$$

being the relative flow velocity.

The constant term in equation (9) is equivalent to the psychrometric equation where the overall rate of evaporation is proportional to the drop diameter; as the Reynolds number becomes large the second term will contribute a term varying with the three halves power of the drop diameter. Since for most gases and vapours the Prandl number is of the order of unity, the second term will be appreciable for Reynolds numbers greater than one. For kinematic viscosities of the order of 2×10^{-1} c.g.s. and drop diameters of 200μ this condition will arise if the relative velocity is greater than one metre per second. Thus it is unlikely that under combustion chamber conditions a linear dependence of rate of burning upon droplet diameter will hold.

It is doubtful whether it is possible for multicomponent propellant systems in general to go further than the statement that the rate of burning will increase, and the time for combustion decrease with decreasing mean drop size of the components. Apparent exceptions may be found even to this generalisation, for instance with reacting bicomponent systems injection in the form of a spray could be disadvantageous compared with simple impingement of the two liquids if the energy released by partial reaction has a more powerful dispersing force than any physical means.

/SECTION II.

SECTION II.

THE ATOMISATION OF LIQUIDS; A REVIEW.

The influence of the physical properties of fuels upon the state of atomisation produced by various injection methods is important, both from the standpoint of injector design, and for the testing and comparison of different individual systems. In many cases it will not be desirable in "cold" tests to use the propellant for which the injector was designed, but to use a liquid which is similar to the propellant in the relevant physical properties or, to adjust other more easily variable parameters in such a way as to give a dynamically similar system.

In the past twenty years a considerable amount of research has been done on the effect of design, operating conditions and the physical constants of the sprayed liquid on spray characteristics, notably by Lee (14) on injectors for Diesel engines, Hottel (15) on spray nozzles for oil furnaces, Nukiyama and Tamazawa (16) on air atomisers, and in England by Shell, Lucas and N.G.T.E. on injectors for gas turbines. A comprehensive review of fuel spray phenomena relating to internal combustion engines has been made by Muraszew (17) and an extensive list of references is given by Roesch and Rose (18).

However, in spite of all this it is clear that the precise relations between the degree of atomisation and the possible variables are still unknown; in view of the apparent complexity of the process, and the technical difficulties of isolating one variable from another, this is not surprising. The necessity for the statistical treatment of results is apparent, since the size of any particular drop is not uniquely determined, but governed by a probability relation; this must be borne in mind when considering the relationships that have been put forward as governing atomisation, and explains the fact that, while there is general agreement on the direction of the variation of mean droplet size with the various parameters, there is very little on the magnitude of the variation. Further, many of the results quoted in the literature are applicable only for the conditions and techniques employed in the original research.

It is, nevertheless, agreed that no fundamental difference exists in the mechanism of the three most common methods employed for the atomisation of liquids, namely, 'solid' or high pressure injection, centrifugal injection, and air or gas atomisation. These differ chiefly in their field of applicability, and the method used for initial dispersion of the liquid. The forces causing atomisation are the inertia of the liquid and the resistance forces between the liquid and gaseous medium. The first process relies on turbulence to spread the liquid into high velocity sheets, so that the resistance forces may complete the process of atomisation, and requires high injection pressures to give the high flow velocities required; the second, as its nomenclature would imply, uses rotational forces to achieve the initial dispersion of the jet, and therefore operates with only moderate pressure heads; the last method imparts the high velocity to the gaseous medium, and needs only a small liquid pressure head; it is most useful for low rates of liquid input, and for combustion systems containing one gaseous component. With correct design and choice of operating conditions, all three methods will give equal degrees of atomisation.

The theory of the break up of liquid jets has been examined by Rayleigh (19) who concluded that for a column of liquid in a state of unstable equilibrium due to the action of capillary forces, the wave length of the disturbance leading most rapidly to disintegration is given by $\lambda = 4.508 \times$ the diameter. This was confirmed by observations on low velocity liquid jets which showed a rotationally symmetric periodic variation in diameter, increasing with increasing distance from the orifice, until individual drops were formed by rupture at the constrictions; the wave length of the disturbance, and therefore the diameter of the drops formed was found to be proportional to the jet radius.

By the use of spark photography, Haenlein (2) discovered that at higher jet velocities the periodic contraction of the jet became rotationally asymmetric, and at still higher velocities, was replaced by a wave-like jet of constant cross-sections, which tended to break at the crests and the troughs through the action of air resistance in increasing the amplitude of the wave; with increase of velocity, a critical value was reached, at which the disruption of the jet into drops appeared to occur at the orifice. Castleman (21) explained the latter process in terms of the Rayleigh theory of the instability of liquid jets by assuming that small portions of the jet surface were drawn out by air resistance forces into fine ligaments, which then broke off from the main surface and contracted by surface tension forces into small drops. This was contrary to current theory, which supposed that high pressure injectors relied on the internal turbulence, and the suddenly released pressure energy of the liquid, to effect atomisation, so that the mechanism was essentially different from that of air atomisation, and would occur even in vacuo.

The experiments of Lee (22) on spark photomicrography of diesel engine sprays showed that Castleman's views on atomisation were essentially correct; he observed that, at low (one thousandth of atmospheric) air densities, the jet is spread out into sheets by orifice turbulence, the influence of orifice conditions and liquid viscosity and velocity becoming very marked; but that the sheets collapse under the action of surface tension forces to give a relatively coarse spray.

As the air density is increased, the stages in drop formation, proposed by Castleman, are observed; the distance from the orifice within which a stable state of atomisation is reached decreases; at normal air densities the effect of increasing density is more on this distance than on the final degree of atomisation, and at high densities the effect of orifice design and fuel viscosity is small, except in so far as they may decrease the initial jet velocity.

Lee studied the spray formation, both by direct photography, and by the collection, measurement and counting of the droplets formed, varying the orifice design, the liquid injection pressure head, the chamber air density and the viscosity and surface tension of the liquid spray. His conclusions were summarised as follows: "When other variables are held constant, the degree of disintegration of the jet (a) increases with the distance from the nozzle, until the disintegrating forces due to the relative velocity between the air and the fuel are no longer sufficient to overcome the resisting forces due to the surface tension and viscosity of the fuel, (b) increases with increase

of air density; (c) increases with increase of jet velocity; (d) decreases with increase of orifice diameter; (e) decreases with increase of fuel viscosity; (f) decreases with increase of fuel surface tension; (g) increases with increase of fuel turbulence. Fuel turbulence accelerates the disintegration of the jet by ruffling its surface close to the orifice, but has relatively little disintegrating effect in itself".

Lee, probably wisely, did not attempt to formulate any precise relationship between a mean drop size and the other parameters; such attempts as have been made by other workers are based upon a somewhat crude dimensional analysis. It is assumed that there are two forces, the inertia of the liquid and air resistance, tending to break up the jet, while viscous and surface forces resist disintegration. These forces may be expressed dimensionally in terms of a velocity, a representative length, the surface tension, viscosity and density of the liquid and density of the gaseous medium (the effect of the viscosity of the gaseous medium is assumed to be negligible). Then;

- (1) the inertia force given by $M \frac{dv}{dt}$ has dimensions $\rho L^2 v^2$
- (2) the air resistance force given by $k \rho_g A v^2$ has dimensions $\rho_g L^2 v^2$
- (3) the viscosity force given by $\mu A \frac{dv}{dL}$ has dimensions $\mu L v$
- (4) the surface tension force given by γL has dimensions γL

In the orifice, and at low air densities, terms (1) and (3) may be taken to be predominant, and the condition for dynamic similarity is that their ratio is constant

Condition A.
$$\frac{L v}{\nu} =$$

In the orifice L can be identified as the orifice radius and condition A implies that the Reynolds number of the flow should be held constant, i.e. an increase in fuel viscosity can be compensated by increasing the flow velocity, maintaining the same type of flow and degree of turbulence in the orifice. In the free jet, L can be taken to be the drop diameter and it has been assumed that, for viscous liquids at low air densities, the most probable drop diameter is directly proportional to the viscosity, and inversely proportional to the jet velocity.

At higher air densities, terms (2) and (4) will predominate, and therefore we obtain

Condition B.
$$\frac{\gamma}{\rho_g L v^2} =$$

This implies that the most probable drop size will be directly proportional to the surface tension, and inversely proportional

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to the air density and the square of the jet velocity. It should be remembered that ratio B is constant only when the Reynolds number with respect to the gaseous medium is high, and the resistance of the medium is independent of the Reynolds number. Treibnigg (23) considered this condition to hold for the maximum possible drop size since, for stability, the average air pressure over the surface of the drop must be balanced by the surface tension pressure. Scheubel (24) modified this condition by introducing the ratio between surface tension and viscous forces, i.e. $d = \varphi \left(\frac{\gamma}{\mu v} \right) \cdot \varphi \left(\frac{\gamma}{\rho v^2} \right)$ but did not check the relationship between drop size and viscosity experimentally.

Sass (25) found that the reciprocal relationship between drop size and air density was approximately true for high air densities, but that there was a smaller rate of increase of drop size with decreasing density at atmospheric densities. Lee (26) and Retel (27) found that the mean drop size goes through a minimum value as the air density is increased, but this conclusion has been criticised on the grounds that, in Retel's experiment, the increase in drop size may be due to decreasing jet velocity, since he was operating with a fixed injection pressure, and in Lee's case the high spray density obtained may have favoured re-combination of the drops. Sauter and Scheubel found the drop size in air atomisers to be approximately proportional to the surface tension, and inversely proportional to the square of the air velocity. Nukiyama and Tamasava (loc.cit) deduced the following empirical formula :

$$\bar{d}_s = \frac{585}{v} \left(\frac{\gamma}{\rho} \right)^{\frac{1}{2}} + 597 \left[\frac{\mu}{(\gamma \rho)^{\frac{1}{2}}} \right]^{0.45} \cdot \left(\frac{1000 Q_l}{Q_a} \right) \quad (10)$$

where Q_l and Q_a are the volume flow rates of liquid and air respectively.

Lewis and Richards (28) have concluded that Sauter's data do not conflict with this equation. They also find that, although as it stands the empirical equation of Nukiyama and Tamasava is not dimensionally homogeneous, with suitable assumptions to obtain the appropriate value of Q_a , it gives the correct order of magnitude of drop size when applied to pressure and centrifugal injectors.

Merrington and Richardson (29) studying the break up of both stationary jets and jets discharged from nozzles having a velocity with respect to the air, found that at high air/liquid velocities the volume medium drop size at a large distance from the orifice was independent of the orifice diameter and of the surface tension of the liquid, and was proportional to the one fifth power of the liquid viscosity, and inversely proportional to the velocity. The range of viscosities studied was 0.004 to 10 Stokes. At low velocities the drop size was dependent only on the diameter of the orifice and not on the properties of the liquid, except in the case of large drops, when break up occurred during the time of fall.

With pressure and centrifugal injectors, the liquid/air
/velocity

velocity is not measured directly, but must be calculated from the liquid density and the pressure drop across the nozzle by means of Bernoulli's equation

$$v = \sqrt{\frac{2 \Delta P}{\rho}} \quad (11)$$

In the case of viscous liquids the actual velocity will be somewhat less than that given by this equation, owing to the dissipation of energy in the orifice. For plain orifices it is possible to calculate the velocity from the volume flow rate

$$Q = k \cdot \text{Area} \cdot \sqrt{\frac{2 \Delta P}{\rho}}$$

where k is the discharge coefficient, and is the product of the velocity coefficient, i.e. the ratio of the actual velocity to that given by Bernoulli's equation, and the contraction coefficient, which is the ratio of the jet area to the orifice area. At Reynold's numbers greater than the critical value (2,000 - 3,000) the contraction coefficient can be taken to be close to unity and the velocity coefficient equal to the measured discharge coefficient. The latter has a usually constant value of between 0.7 - 0.9 above the critical Reynolds number, the actual value depending upon the orifice length/diameter ratio and fluid viscosity.

With centrifugal nozzles the contraction coefficient is usually less than unity, due to the presence of an air core in the orifice, and is a function of the viscosity. It has been shown by Watson (30) that, neglecting viscosity, the discharge coefficient can be calculated from the ratio of tangential to axial velocity imparted to the liquid; and therefore from the design of the nozzle. Thus it might be possible to estimate the velocity losses from the ratio of the actual to the calculated discharge coefficient.

However, Taylor (31) has shown that, even for liquids of kinematic viscosities of the order of that of water, the major part of the liquid flow forms a boundary layer, and it would appear from his calculations that constant discharge coefficients and similarity of flow will only obtain when the Reynolds number is kept constant. In view of this, it is not easy to compare the results of investigators on centrifugal nozzles, where the drop size is represented as a function of the pressure drop across the nozzle with the equations relating drop size to relative velocity and other parameters. Hottel and Longwell, using centrifugal nozzles having cone angles between 60° and 120° with burning oils having viscosities between 0.1 and 0.9 stokes at injection pressures between 50 and 300 pounds per square inch, found that their results could be expressed as:

$$\frac{d_0 \sin \frac{\alpha}{2}}{r_0} = \frac{e^{0.705 V}}{\Delta P^{0.375}} \quad (12)$$

d_0 being the mean drop size defined by the Rosin Rammelin equation

$$R = e^{-\left(\frac{d}{d_0}\right)^n}$$

r_0 the radius of orifice and α is the spray cone angle. The latter is itself dependent upon the viscosity, decreasing by about 20% over the range of viscosities studied. The introduction

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of the term $\sin \frac{\alpha}{2}$ which helps to linearise the dependence of drop diameter on orifice radius is justified by Longwell on the grounds that the thickness of the conical sheet of liquid, formed at a distance l from the orifice along its surface, is proportional to $t_0(1 - \frac{l}{r_0}) / \sin \frac{\alpha}{2}$

where t_0 is the thickness at the orifice. Then, if it is assumed that the size of the drops formed by break up of the sheet is proportional to its thickness, and if t_0 is very much less than r_0 , i.e. if the discharge coefficient is small, then

$$\frac{d_0 \sin \frac{\alpha}{2}}{t_0} \sim \frac{l}{r_0}$$

i.e. inversely as the ratio of the length of the liquid sheet to its thickness at the orifice. This ratio is assumed by Longwell to be independent of the scale and design of the nozzle, and to be a function only of the injection pressure and liquid viscosity. However, if the initial thickness is independent of pressure, as it should be if the discharge coefficient is constant, the length of the conical sheet should increase with increasing pressure to effect a decrease in drop size. This is not borne out by observation, for whereas the pressure increases, the point of break up of the sheet approaches the orifice. Work in England has led to the conclusion that the drop size is a minimum when the cone angle is about 90° and is not linearly proportional to the orifice size. Needham (32) has concluded that, for a number of injectors of this optimum angle, the drop size can be expressed by

$$d_0 = c \cdot \frac{Q_L^{0.25}}{\Delta P^{0.4}} \quad (13)$$

where the pressure range covered was 6 to 125 pounds per square inch,

$$\text{since } Q_L = k \pi r_0^2 \sqrt{\frac{2 \Delta P}{\rho}} \quad (14)$$

$$d_0 = c \cdot \frac{r_0^{0.5}}{\Delta P^{0.275}}$$

This apparent divergence of opinion on the dependence of drop size on orifice size and injection pressure is no doubt due in part to different methods of treating the data, but it is indicative of the difficulty of formulating a generalised theory of atomisers of even one type, the centrifugal nozzle.

For the testing and comparison of rocket injectors, it is clear that there is not yet enough information available on the effect of the physical properties of liquids on drop size to permit the simulation of one liquid by another, allowing for the change in one property by adjustment of another. If, for reasons of convenience, it is desired to use a substitute liquid each relevant physical property should be reproduced as closely as possible. In so far as the properties of the gaseous medium into which the liquid is sprayed are concerned simulation is not possible. In the combustion chamber the gas density may be from two to four times that of air at atmospheric pressure.

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The mean gas temperature may lie between 1000 and 2000°A, therefore the gas viscosity will be some four to eight times larger. The changes in the kinematic viscosity will be rather less owing to the increased density. The effect of the increased temperature of the liquid on its viscosity and more particularly on its surface tension must be considerable. The surface temperature of the liquid will be equal or near to the boiling point under the combustion chamber pressure; this may approach the critical temperature at which the surface tension vanishes.

The flow condition in the gaseous medium in a combustion chamber will differ considerably from those of an unignited spray in the open air. It has been observed that the shape and distribution of sprays are modified by the shape and size of the chamber. Further, under conditions of forced evaporation the velocity of flow normal to the surface of the liquid may be comparable to the tangential velocity between the liquid surface and the gaseous medium; this will reduce the resistance forces tending to break up the drop.

It is not possible to assess the overall effect of these conditions on the atomising properties of an injector and there seems to be a wide field for experimental research into the functioning of injectors under conditions approaching those in the combustion chamber.

/SECTION III.

SECTION III.

THE MEASUREMENT OF SPRAY DROPLET SIZE; A REVIEW.

It is obvious from the preceding discussion that, while knowledge of the drop size distribution of injector sprays is desirable in so far as size distribution must affect the efficiency and extent of the combustion, and while it will be profitable to attempt to correlate this information with proofstand tests, a high degree of accuracy is not required. On the other hand in view of the uncertainty as to the order of relationship (quantitatively) between drop size and combustion characteristics it will be desirable to measure the size distribution rather than any mean parameter such as specific surface.

Bearing these facts in mind, it is useful to review briefly the methods, both direct and indirect, which have been used to measure drop sizes in sprays. In the former the droplets in a representative sample of the spray are caught, measured and counted, while indirect methods rely on the measurement of a property of the spray which can be related to some mean dimension. The advantage of direct methods is the ease with which distribution curves can be obtained from the data. The disadvantages are the excessive time and labour required for the assessment of the spray, technical difficulties in catching the spray in a form suitable for measurement, and the sampling problems which arise for non-homogeneous sprays.

On the other hand indirect methods in general do not give any information on the dispersion of drop sizes about the mean value, require considerable ingenuity for application and are liable to somewhat large instrumental errors; but results can be obtained without the expenditure of excessive time and labour after the initial setting up of the apparatus.

The direct methods of spray droplet size determination used by previous investigators (33) differ mainly in the means adopted to catch samples of the spray in a form suitable for subsequent counting and measurement of the individual droplets. The simplest method is to expose a glass microscope slide to the spray for such a time as to give a convenient density. If an uncoated slide is used, the liquid drops wet the glass surface forming plano convex lenses of which the volume can be estimated from their diameter and focal length. This method is tedious and can only be successfully applied to non-volatile liquids; further the droplets must not exceed a certain size or they will tend to break up on impact with the slide. A simple method which can be applied to large drops is to catch them on sheets of absorbent paper, the liquid is preferably dyed so that the stains, which can be previously calibrated using drops of known size, are easily observed (29).

Slides coated with a layer of viscous liquid in which the droplets embed themselves, retaining their spherical shape, have been used for both oily and aqueous sprays, gelatinised glycerine and mixtures of paraffin jelly and paraffin oil respectively being employed. If the medium is previously

/saturated

saturated with the sprayed liquid and the slide covered with a cover slip after exposure, the records are stable over a period of several days and even the smallest droplets can be observed.

Surfaces coated with carbon and magnesium oxide smoke will show indentations due to the impact of droplets which may be related to the size of the droplets by previous calibration. Slides coated with a soluble dye have been shown to be sensitive to droplets of the order of 1μ . Dobie (34) has used dishes containing castor oil diluted with alcohol to give a density similar to that of the sprayed liquid and has also caught the droplets on photographic plates coated with an oil and vaseline mixture obtaining a permanent record by subsequent exposure to light, removal of the oily film with ether and development of the light sensitive emulsion.

These methods need some form of mechanical shutter between the spray and the slide if a density, small enough to prevent coalescence of the drops upon the slide, is to be obtained. With dense sprays and short times of exposure there will be a danger of break up of drops upon the edges of the shutter, and with droplet diameters of the ranges existing in injector sprays there is a high probability of break up of the larger drops on impact with the slide. Although the method lends itself to the simultaneous determination of variations of spray density and droplet distribution at various planes in the spray volume it is not considered that it is of general application for injector testing.

An ingenious method of catching the spray in a form suitable for either direct assessment or for the indirect determination of surface area has been used by Longwell in America and by the Shell Laboratories in England. The former has collected oil sprays in a freezing bath of solid carbon dioxide dissolved in alcohol; the oil droplets solidify and the particle size distribution is obtained by fractional sieving; the use of low temperatures has obvious disadvantages on the experimental side. Joyce (5) has examined the sprays produced by gas turbine injectors by substituting for the fuel used operationally, a suitable grade of paraffin wax heated to such a temperature that it has physical properties similar to the fuel. The liquid wax spray solidifies in flight and the wax particles are collected in water. They may then be fractionally sieved, or measured and counted on a microscope slide (35). Successful attempts have been made to suspend the particles in a suitable liquid and determine the specific surface by measuring the optical transmission of the suspension and it is possible that the size distribution curve could be obtained by sedimentation (36).

This method has the considerable advantage that the sample obtained is representative of the whole spray, and with the sieving method can be made much larger than is possible with methods involving counting. The validity of the assumption that the solid paraffin wax spray is identical with that given by liquid oil is open to question: it will depend on the rate of cooling of the spray and the time taken for a stable state of atomisation to be reached. Hollow wax droplets have been observed in wax sprays, due probably to the slight solubility of the pressurising gas in the liquid wax, and it would seem desirable that a check against a method of known accuracy, using

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both liquid wax and the operational liquid, should be made before the method can be accepted as reliable. It is not very suitable for sprays of high droplet density where partial coalescence of the wax spheres tend to occur, and it is difficult to see how this method can be applied to injectors designed to operate with liquids resembling water in their physical properties, or for impinging jet atomisers.

The difficulties inherent in the application of the above methods to injector sprays have led to the study of the possibilities of photography of sprays in flight, using high intensity short duration illumination to "freeze" the droplets in space. Although spark photography has long been used to obtain information on the mechanism of atomisation, it does not seem to have been much used for the measurement of droplet sizes, probably because the demands for high resolution and large depth of field are irreconcilable. However, the advantages in flexibility of application of this method are considerable. In distinction to most other methods, there is no interaction between the subject and the method of observation; it can be applied to any point in the spray; in rocket injectors considerable interest may be centred on the unstable states of dispersion close to the orifice or at the point of impingement of jets. Providing transparent windows can be used, it can be applied to atomisation under conditions other than atmospheric, and in high velocity gas streams. Simultaneously with drop size determination it will give information on the spray density at any point and with slight modification on the droplet velocities. The main non-technical disadvantage of the method as applied to injector assessment lies in the tendency to focus attention on the individual droplet at a certain point in space and time, whereas the time average over the total spray volume is more relevant. This necessitates care in sampling inhomogeneous or intermittent sprays.

The experimental difficulties lie in the use of ultra short exposure times to prevent blurring of the image of the drop by its motion relative to the camera and obtaining adequate resolution without excessive limitation of the depth of field. The first is not a serious problem at the present time; for a drop moving with a velocity of ten metres per second an exposure of 1 micro second will limit the movement during exposure to 10 micron; since a limit of resolution of the order of 20 micron is adequate, the exposure time of one or two microseconds, given by standard 'Arditron' equipment, should be satisfactory for most sprays.

The sprays encountered in injector research contain droplets having diameters of between 50 and 1000 micron; the weight fraction of spray outside this range is small and for any particular spray the bulk is contained in a ten or twenty fold diameter range. Thus for convenience in measurement a ten to fifty fold linear magnification will be required.

For several reasons it is desirable to perform this magnification in two stages, using a magnification of about two on the photographic plate and obtaining the desired degree of enlargement by subsequent projection. The illumination of the image is related to brightness of the source B, the numerical aperture of the objective N.A; and the degree of magnification M by the relation:

$$I = c \cdot \frac{B(N.A)^2}{M} \quad (15)$$

The level of illumination required is determined by the speed of the emulsion used, and to obtain high contrast this is kept low. In view of the short exposures needed the source brightness used is the maximum obtainable. In order to obtain maximum depth of field the numerical aperture is made small subject to the limitation of diffraction effects; thus it is necessary to restrict the initial degree of magnification. A lower limit for this is fixed by the resolution and grain size of the emulsion, the former for process emulsions is about 10μ , and although the required degree of resolution is unlikely to be less than 20μ it is undesirable to have an initial magnification of less than unity. Further, the projection distance becomes inconveniently long if second stage magnifications of more than fifty are required.

The depth of focus is determined by the degree of resolution required; the indeterminacy in the measurement of the diameter of a spray droplet will be approximately equal to the resolving power, i.e. the least distance between two object points at which they may be distinguished. For a spray of weight median diameter of two hundred micron, a resolution of twenty micron is sufficient. The depth of field is related to the circle of confusion z , and the numerical aperture N.A. by the relation:

$$d = \frac{z}{M.(N.A)} \quad (16)$$

while a relation exists between z^1 , the limit of resolution, N.A. and the wave length of the illumination λ , representing diffraction by the aperture:

$$z^1 = \frac{1}{2} \frac{\lambda}{(N.A)} \quad (17)$$

There is no object in using a circle of confusion z less than Mz^1 , therefore, eliminating N.A. from equations 16 and 17.

$$d = \frac{2(z^1)^2}{\lambda} \quad (18)$$

Thus, if z^1 is put equal to 20μ and $\lambda = 0.5\mu$ the maximum depth of focus will be 1.6 m.m. Thus a given degree of resolution is obtained only over a relatively small depth of the spray. This depth is independent of the optical parameters of the lens system, and is dependent only on the wave length of the illumination. It is impracticable to decrease this other than by using a blue sensitive emulsion having a maximum sensitivity at 0.45μ .

In view of the lack of information on the practicability of the photographic method, experimental work has been started at E.R.D.E. During early experiments an attempt was made to limit the depth of field photographed by restricted illumination at right angles to the camera axis. The droplets were recorded by the light reflected into the camera lens. However, no short duration source of sufficient brightness was found, and it was decided to concentrate work on axial illuminating systems, the drops being silhouetted against the light field. The use of slits to limit the depth of spray in the camera field was examined, but although it was found possible to prevent break-up

/of

of the spray on the edges of the slits by making them of absorbent material and limiting the time of exposure to the spray by means of a shutter, it was decided that the introduction of mechanical devices into the spray would remove the special advantages of the photographic method.

When a droplet field of a depth large compared to the depth of field of the camera system is photographed, two sources of error are introduced. Outside the focus zone the definition falls off gradually, so that in the assessment of the spray photograph the operator has to judge the point at which measurement becomes impracticable. As a result, the large droplet sizes are estimated over a greater depth of the spray than the smaller droplets and the small droplet fractions are under-weighted. For a given error in estimation of droplet diameter, the fractional volume error will be proportional to the reciprocal of the droplet diameter. On the other hand, there is a tendency to under-estimate the diameter of drops out of focus the error increasing with lack of sharpness of the image.

The distortion introduced into the distribution curve, due to these effects, was examined by photographing a suspension of paraffin wax spheres in water contained in parallel sided glass absorption cells of 1 and 2 cm. thickness. The distribution curve of the wax suspension had been previously determined by counting and measuring a sample taken on a microscope slide. With the cell of 1 cm. thickness, the distribution curve obtained was not significantly different from that given by the slide count. With the 2 cm. cell the apparent weight median diameter had decreased by approximately 20μ , and the slope of the distribution curve showed a slight increase. It was concluded that the accuracy of the method was sufficient for sprays of size constant greater than 200μ . With sprays of higher dispersion the method is comparative rather than absolute. The assessment of the photographic record of the spray is best performed by projecting the image on to a ground glass screen. The counting and measurement of a sufficient number of drop images is somewhat laborious since more than five hundred must be counted in order to obtain reproducible results. It would be possible to devise a photoelectric scanning device operating on the negative which would sort and count the drops in a number of size ranges with sufficient accuracy, but the time and labour required to design and construct this would be prohibitively large. Manual measurement and counting can be speeded up by the use of an adjustable comparison scale, the movement of which is mechanically connected to a multi-position switch selecting the appropriate counting register. A counting aid operating on this principle is described in Part II of this report.

It should be possible, by measuring the mean transmission of the negative, to obtain a value for the total surface area of the spray sample photographed. The problems arising will be considered in the review of other methods of indirect assessment of sprays.

Indirect methods rely on the measurement of some property of the spray which can be related to a mean diameter. The method which would seem to be most applicable to injector sprays is the measurement of the optical transmission of the spray. Then, for a spray density and path length such that over-lapping of the droplets in the direction of the optical axis can be neglected:

$$T = 1 - \frac{S}{4V} \cdot \lambda l = 1 - \frac{3\lambda^2}{2d_s^2} \quad (19)$$

/assuming

assuming the droplets are opaque. Here S/V is the surface area per unit volume of liquid, v is the volume concentration of liquid, and l is the optical path length. If T is less than 0.8, the error due to overlapping of the droplets cannot be neglected, and the exponential equation

$$T = e^{-\frac{3v\ell}{2}} \quad (20)$$

must be used. This holds only over path lengths within which the values of v and d_s are constant. Thus it can be applied only to homogeneously distributed sprays; otherwise the overall transmission must be greater than 0.8. This limits the possible applicability of the method to injector sprays to those of low density or spatial homogeneity. With values of the transmission coefficient greater than 0.9 the error in d_s for a given error in the transmission coefficient becomes considerable (it is nine times the error in transmission coefficient at $T = 0.9$). The measurement of the ratio of the volume of liquid to the volume of spray presents some difficulties. Sauter (37), using the method to investigate air atomisers, assumed the volume concentration of liquid to be equal to the ratio of the volume flow rates of liquid and air, thereby assuming the velocity of the air and the liquid to be identical.

In the case of pressure and centrifugal injectors, the quantity most susceptible to measurement is the volume flow rate across unit surface at any point in the spray. To derive the concentration from the flux density, the mean velocity of the spray normal to the surface must be known or assumed. This necessity for measurement of spray velocities renders the method inconvenient for the routine testing of pressure and swirl injectors.

Sauter also used the charge carried by drops emitted from a nozzle held at a fixed electrostatic potential to measure the mean diameter with respect to surface; the experimental difficulties make this method of historical interest only. A method which might be employed as a rough check upon the diameter and homogeneity of the sprays relies on the diffraction of light from a point source observed through the spray. If monochromatic light is used, and the spray is homogeneous, a series of concentric diffraction rings surrounding the source can be observed. The half angle subtended at the eye by the n th dark ring is related (38) to the drop diameter d and the wave length λ by the relation:

$$d = \frac{\lambda(n+0.25)}{\sin\theta_n} \quad (21)$$

The definition of the rings deteriorates with increasing inhomogeneity.

/SUMMARY.

SUMMARY.

The text is divided into three sections, the first of which is a discussion of the relation of propellant atomisation to combustion in rocket motors, the second is a review of the literature on the atomisation of liquids and the third a review of methods of measuring drop size in sprays. The reason for this order of presentation is that the method eventually selected for assessing the dispersive power of liquid propellant injectors will be determined by the importance of atomisation in the combustion process of the rocket motor and by the variation of droplet size with changes in the physical properties of the propellant and the external conditions.

A spray consisting of a number of spherical droplets of different sizes can be described by two quantities, one representing a mean droplet diameter and the other the 'spread' of droplet size about this mean diameter. It has been found experimentally that the distribution of droplet size in the sprays given by some types of atomisers can be represented by the Rosin-Rammler expression:

$$R = e^{-\left(\frac{d}{d_0}\right)^n}$$

This equation defines the 'size constant', d_0 , and the distribution constant, n . It is shown that if a relation between the rate of combustion of a droplet of propellant and its diameter can be found the combustion characteristics of a spray can be defined by d_0 , n , and the droplet diameter constants in the rate of combustion equation. By analogy with the combustion of solid propellants it might be thought that the rate of combustion of a droplet of a monopropellant would be proportional to the surface area, that is to the square of the diameter of the droplet. This conclusion is shown to be true only when the radius of curvature of the droplet surface is small compared with the dimensions of the preheating zone which must exist in the vapour phase surrounding the droplet. In theory the variation of rate of combustion can be calculated from the physical-chemical constants of the propellant system; in practice the calculation is laborious and the value of the constants of any particular system uncertain. An approximate treatment is attempted which indicates that for droplets of the order of diameter to be expected in the combustion of liquid propellants in rocket systems the rate of combustion per drop will be proportional to some power of the diameter between one and two.

$$\text{i.e. } \dot{M} = k'_\beta d^\beta, \quad 1 \leq \beta \leq 2$$

The value of β can be estimated if the rate of burning per unit area of plane surface of the monopropellant is known. As this increases β tends to its limiting value of two.

In the case of bipropellant systems it is impossible to make any estimate of the value of β without a detailed examination of the mechanism of the process of combustion for particular systems. The influence of changes in the state of gas flow in the combustion chamber will make the experimental study of the influence of droplet size more difficult than in monopropellant

/systems.

systems.

Assuming that the rate of combustion per drop is given by $\dot{M} = k_{\beta} d^{\beta}$ and that the distribution of droplet sizes in the spray can be represented by $R = e^{-(d/d_0)^n}$ the initial rate of combustion of the spray per unit volume of propellant is given by

$$\dot{M}_v = \frac{k_{\beta}}{d_0^{3-\beta}} \left[\left(1 - \frac{3-\beta}{n} \right) \right]^{\frac{1}{3-\beta}}$$

This expression indicates that the initial rate of combustion of the spray increases with decreasing size constant and decreasing homogeneity (decreasing values of n).

Under steady operating conditions the overall rate of combustion must be proportional to the rate of input of propellant (and equal to the latter for maximum efficiency). The volume of propellant in the combustion chamber is given by

$$V_{c.c} = \frac{3-\beta}{6-\beta} \cdot \frac{\dot{V} d_0^{3-\beta}}{k_{\beta}} \cdot \left(1 + \frac{3-\beta}{n} \right)$$

k_{β} being defined by the equation for the variation of droplet diameter with time:

$$d^{3-\beta} = d_i^{3-\beta} - k_{\beta} t$$

$V_{c.c}$ increases with increasing d_0 indicating that if the density of combustion is kept constant the volume of the combustion chamber must increase with increasing d_0 ; on the other hand it is not very dependant on the value of n . The fraction of propellant unburnt at a time t after injection is more dependent upon the inhomogeneity of the spray. It can be expressed as a function of the parameter $k_{\beta} t / d_0^{3-\beta}$. At a value of t which makes this expression unity a change of n from 4 to 2 increases the fraction of propellant unburnt by a factor of 2 for $\beta=1$ and by a factor of 5.8 for $\beta=2$.

This formal treatment indicates the direction of the effect of changes in the fineness and inhomogeneity of the propellant spray. The uncertainty in the appropriate value of the index in the rate of burning equation and the simplifying assumptions used in this analysis make more precise deductions fruitless.

The next section on the process of atomising reveals a similar uncertainty in our knowledge. It is clearly true that the fineness of a spray increases with decreasing density, surface tension, and viscosity of the propellant, and decreases with increase of orifice size and decrease of injection pressure drop or combustion chamber gas density. It is difficult to assess the exact state of the atmosphere in which the atomisation takes place in a rocket motor and the effect of this state upon the process of atomisation. The temperature and flow conditions are quite unlike those encountered in tests in which the spray is not ignited. It is probable that the dispersion is assisted by the high temperature and that the resultant spray is finer than that given by the same injector in 'cold' tests. The process of atomisation is not instantaneous, and, since it takes place during combustion, may never be complete. It seems desirable to

/investigate

investigate this process under combustion conditions.

Clearly a high degree of precision will not be needed in the assessment of the atomising properties of rocket injectors. The requirement is for a method, flexible in application, which will give at least a semi-quantitative measure of the distribution of droplet sizes and which can be applied to the incipient stages of atomisation as well as to the fully developed spray. For these reasons after reviewing the methods which have been used for droplet size measurement we have concentrated on photographic means of recording the spray in flight. It is shown that this method is capable of greater accuracy than is actually required both in the estimation of the diameter of an individual droplet and in the measurement of the distribution curve of the spray as a whole. It is particularly suited to the study of the unstable state of the spray close to the injector orifice and of atomisation under conditions of high temperature, high air density etc. The theory of the photography of sprays is discussed in detail. The technique and apparatus developed at E.R.D.E. is described in Part II with the experimental results obtained for representative injectors. It is intended to correlate tests on injectors giving widely varying mean droplet diameters with proofstand measurements on monopropellants.

The majority of the problems raised have not been solved. It is hoped however, that it will serve to stimulate interest and further research.

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/APPENDIX A.

APPENDIX A.

THE USE OF THE ROSIN-RAMMLER DISTRIBUTION LAW IN THE REPRESENTATION OF PROPELLANT SPRAYS.

In dealing with size distribution of uniformly shaped particles one is usually interested in the volume fraction of the distribution having dimensions between any two arbitrary limits rather than in the number of particles in this size range; particularly so when the form of the distribution is obtained by sieving methods and the method of presentation of distribution curves is a plot of the fraction or percentage of the volume having a diameter greater than a length d , against values of d . From observations on powdered coal Rosin and Rammler noted that the distribution curves could be made linear by plotting the double logarithm of the reciprocal of the volume fraction oversize, R , against the logarithm of the diameter d . This indicates that the distribution curve can be expressed as :

$$R = e^{-bd^n}$$

$$\text{or, } \log.\log \frac{1}{R} = \log b + n \log d + \log.\log e$$

n being the slope of the straight line obtained. Since n defines the 'spread' of the points it is termed the distribution constant. It is possible to define the constant b in terms of the volume median diameter, the diameter above which lies one half the total volume, but it is mathematically neater to define a diameter d_0 by the equation:

$$R = e^{-\left(\frac{d}{d_0}\right)^n} \quad (1)$$

Then $1/e^{\text{th}}$ of the volume has a diameter greater than d_0 . This expression has been found to fit the droplet distributions existing in centrifugal sprays, and as cited (P.10) has been used in conjunction with a relation between the rate of evaporation and the diameter of volatile droplets to deduce certain conclusions about the burning of oil sprays. The distribution constant, n , which defines the slope of the distribution curve has been found to take values between two and four for oil sprays produced by swirl atomisers. It is useful to examine the effect of variation of n on the volume and numerical distribution. The variations of the volume fraction oversize at various arbitrary values of d/d_0 are tabulated below:

	d/d_0							
	0.1	0.2	0.4	0.8	1.2	1.6	2.0	2.4
$n = 2$	0.990	0.961	0.852	0.527	0.237	0.077	0.018	0.003
$n = 2.5$	0.997	0.982	0.905	0.570	0.206	0.039	0.004	0.000
$n = 3.0$	0.999	0.992	0.938	0.599	0.177	0.007	0.000	0.000
$n = 3.5$	1.000	0.997	0.961	0.640	0.144	0.006	0.000	0.000
$n = 4.0$	1.000	0.998	0.975	0.664	0.126	0.001	0.000	0.000

Thus for sprays having distributions of this type, the volume
/fraction

fraction having diameters outside the ranges $0.1 d_0 < d < 2.4 d_0$ for $n = 2$ and $0.2 d_0 < d < 1.6 d_0$ for $n = 4$ is negligible.

Putting $d/d_0 = X$ and differentiating equation (1) with respect to X , we obtain

$$dR = -nX^{n-1} e^{-X^n} dX \quad (2)$$

dR is the volume fraction between X and $X + dX$. Putting this equal to $v_X dX$, v_X being the fractional volume distribution function and differentiating again

$$\frac{dv_X}{dX} = -nX^{n-2} e^{-X^n} (n-1 - nX^n) \quad (3)$$

Equating (3) to zero, it follows that the maximum volume of the spray occurs at a diameter given by :

$$X = \left(\frac{n-1}{n} \right)^{\frac{1}{n}} \quad (4)$$

It can be seen that the volume distribution curve has a maximum at a real value of X provided that $n > 1$. From equation (2) the same condition holds for v_X to vanish at $X = 0$; this implies that the distribution law (1) applies to real distributions only for values of n greater than one.

The distribution functions of other parameters of the system can be derived from equation (2) e.g.

$$S_X \text{ the surface distribution function} = \frac{v_X/X}{\int_0^\infty \frac{v_X dX}{X}} = \frac{6v_X}{S_V d_0 X}$$

$$L_X \text{ the diameter distribution function} = \frac{v_X/X^2}{\int_0^\infty \frac{v_X dX}{X^2}} = \frac{6v_X}{\pi L_V d_0^2 X^2}$$

$$N_X \text{ the number distribution function} = \frac{v_X/X^3}{\int_0^\infty \frac{v_X dX}{X^3}} = \frac{6v_X}{\pi N_V d_0^3 X^3}$$

S_V , L_V , N_V being the total surface, diameter and number respectively per unit volume. The integral in the denominator of each expression can be evaluated by its relation to the gamma function

$\Gamma(1+X)$ which is identical with the generalised factorial function $(x)!$. Values of this function can be found in Jahnke-Emde (42).

Since from equation (2) $v_X = -nX^{n-1} e^{-X^n}$

$$\text{then } \int_0^\infty \frac{v_X}{X^\alpha} dX = \int_0^\infty -nX^{n-\alpha-1} e^{-X^n} dX$$

/It

It can be shown that $\int_0^\infty X^\alpha e^{-X^n} dX = \frac{(\frac{\alpha+1}{n})!}{\alpha+1}$, $\alpha > -1$

it follows that $\int_0^\infty n X^{n-\alpha-1} e^{-X^n} dX = (\frac{n-\alpha}{n} - 1)!$

provided that $n > \alpha$

By differentiation of X^α/X^n with respect to X it can be shown that it has a maximum at a value of X given by $X = (\frac{n-\alpha-1}{n})^{1/n}$

When $n < \alpha+1$ the distribution function corresponding to the value of α has no maximum for real values of X and is finite at $X = 0$. When $n < \alpha$ the distribution function is infinite at $X = 0$ and the definite integral (8) cannot be evaluated. If the total value of the parameter i.e. S, L, N etc. in (5,6,7) or cumulative values are required for $n < \alpha+1$ it is necessary to impose a lower limit on X . Then in the range $\alpha < n < \alpha+1$ the integral can be evaluated from tables of the incomplete gamma function and for $n = \alpha$ from values of the exponential integral; for $n < \alpha$ numerical integration is necessary. If the rate of combustion etc., per drop is given by $k_\beta d^\beta$, the rate of combustion per unit volume is $6k_\beta/\pi d^{3-\beta}$ then in the notation of P.3, total rate over all droplet sizes is

$$\dot{M} = k_\beta \frac{6}{\pi} \sum_{i=1}^N \frac{v_i}{d_i^{3-\beta}} \quad (11)$$

and the mean diameter \bar{d}_β for this dependence of rate upon diameter is defined by the equation :

$$\text{Overall rate of combustion} = 6k_\beta \frac{V}{\pi \bar{d}_\beta^{3-\beta}} \quad (11a)$$

$$\text{thus } \bar{d}_\beta = \left[\sum_{i=1}^N \frac{v_i}{d_i^{3-\beta}} \right]^{\frac{1}{\beta-3}} \quad (12)$$

When $\beta = 1$, this represents the mean diameter considered by Probert and when $\beta = 2$, it is identical with the Sauter mean diameter.

From equation (9)

$$\bar{d}_\beta = \left[d_0 \left[1 - \frac{3-\beta}{n} \right] \right]^{\frac{1}{\beta-3}} \quad (13)$$

when the summation is replaced by integration over all possible values of droplet diameter ranging from zero to infinity. As $n \rightarrow 3-\beta$ the denominator of equation 13 and therefore the rate of combustion per unit volume tends to infinity, and it is necessary to assume a lower limit of droplet size.

/If

$$\text{If } \beta = 1 \quad \bar{d}_1 = d_0 \left[\left(1 - \frac{2}{n} \right) \right]^{-\frac{1}{2}} \quad (14)$$

we can compare the function with that derived from

$$\bar{d}_1 = d_0 \left[\int_{X_1}^{\infty} n X^{n-3} e^{-X^n} dX \right]^{-\frac{1}{2}} \quad (15)$$

for values of n between 2 and 3, and X_1 , an arbitrary limit.

If we put $y = X^n$ in the integral of equation 15, it becomes

$$\int_{y_1}^{\infty} y^{-\frac{2}{n}} e^{-y} dy \quad (16)$$

$$\begin{aligned} \text{or if } n > 2, \quad & \int_0^{\infty} y^{-\frac{2}{n}} e^{-y} dy - \int_0^{y_1} y^{-\frac{2}{n}} e^{-y} dy \\ = \quad & \left(-\frac{2}{n} \right)! - \left(-\frac{2}{n}, y_1 \right)! \end{aligned} \quad (17)$$

This expression, the incomplete factorial function, can be found from tables; these give values of $(x, y)! / (x)!$

then equation 17 is best formulated as

$$\left(-\frac{2}{n} \right)! \left[1 - \frac{\left(-\frac{2}{n}, y_1 \right)!}{\left(-\frac{2}{n} \right)!} \right]$$

when $n = 2$, the integral reduces to the exponential integral

$$-Ei(-x) = \int_x^{\infty} t^{-1} e^{-t} dt$$

values of which are tabulated.

In the calculation of cumulative volume curves from the measurements of the number of drops having diameters between arbitrary limits, the error due to the finite width of the diameter increment may be appreciable when the diameter range is divided into less than ten increments. The cumulative fraction oversize is calculated from the equation:

$$R = 1 - \frac{\sum_0^d N_i \bar{d}_i^3}{\sum_0^{\infty} N_i \bar{d}_i^3} \quad (18)$$

N_i being the number of drops having diameters between d_i and $d_i + \Delta d_i$. The mean diameter \bar{d}_i is taken to be the arithmetic mean of the limiting values when Δd_i is appreciable compared to d_i . The error due to this assumption has been examined by comparing the computed curves with an original cumulative curve obeying the Rosin-Rammler formula.

/From

From equation (7) the fraction of the total number of droplets between diameters d_1 and d_2

$$\text{is } \int_{X_1}^{X_2} \frac{v_x dX}{X^3} / \int_0^{\infty} \frac{v_x dX}{X^3} \quad X_1 \text{ being } \frac{d_1}{d_0} \text{ etc.} \quad (19)$$

From equations (2) and (9) this is equal to

$$\left[(1 - \frac{3}{n}, X_2)! - (1 - \frac{3}{n}, X_1)! \right] / (1 - \frac{3}{n})! , n > 3$$

This can be written as $I(u, p) - I(u_1, p) *$

$$u = \frac{X^n}{\sqrt{1 - \frac{3}{n}}} \quad \text{and} \quad p = -\frac{3}{n}$$

Given the values of X_1 etc., and n , this can be evaluated from tables of the incomplete gamma function. *

Taking $n = 4$, so that $p = -0.75$ and $u = 2X^4$

X	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7
u	0.002	0.016(2)	0.125	0.480	1.31(2)	2.92(8)	5.712	10.12(5)	16.704
$I(u, p)$	0.11034	0.33044	0.54485	0.73747	0.89143	0.96658	0.99404	0.99954	0.99999
$\Delta I(u, p)$	0.11034	0.22010	0.21441	0.19261	0.13396	0.08172	0.02913	0.00550	0.00045
$\overline{\Delta X^3}$	-	0.008	0.064	0.216	0.512	1.0	1.728	2.744	4.096
Δv	-	0.00618	0.04816	0.14601	0.27666	0.28682	0.17669	0.05300	0.00649
R	1.0	0.9938	0.9456	0.7997	0.5230	0.2362	0.0595	0.0065	0.0000
e^{-X^4}	0.9999	0.9919	0.9394	0.7866	0.5169	0.2322	0.0578	0.0061	0.0002
$\delta \times 10^4$	1	19	162	131	61	40	17	4	2

The last line gives the difference between values of the cumulative volume fraction derived from

$$R = e^{-X^4}$$

and the values computed for finite intervals of width $0.2X$ using the arithmetic mean of the interval for the calculation of the volume fraction.

If we consider the injection of a propellant spray into a combustion chamber

/of

* See "Tables of the Incomplete Gamma Function". Karl Pearson, Stationery Office, 1922.

of unlimited size at a rate \dot{V} volumes of propellant per second, a steady state will be reached in which the volume and droplet size distribution will be such that the rate of combustion is balanced by the rate of input. This volume and distribution will depend upon the burning characteristics of the propellant and can be calculated from this and a knowledge of the distribution of the spray at the moment of injection.

If the mass rate of combustion per drop is proportional to a power β of the diameter then the rate of change of diameter with time is proportional to $d^{3-\beta}$. By integration it follows that if d_i is the diameter at $t = 0$ the diameter at time t is given by

$$d^{3-\beta} = d_i^{3-\beta} - k_\beta t$$

where k_β is a constant (20)

and provided that $\beta \neq 3$

Then the ratio of the volume of spray made up of drops initial diameter d_i at time t to its initial volume is

$$\left(1 - \frac{k_\beta t}{d_i^{3-\beta}}\right)^{3/3-\beta} \quad (21)$$

If V_i is the volume distribution function of the injected spray at $d = d_i$ then $\dot{V}_i V_i \delta d \delta t$ is the volume between d_i and $d_i - \delta d$ injected in time δt and the volume in the combustion chamber due to the injection of drops size of this range is

$$\int_0^{t_i} \dot{V}_i V_i \delta d \left(1 - \frac{k_\beta t}{d_i^{3-\beta}}\right)^{3/3-\beta} dt = \frac{3-\beta}{6-\beta} \frac{\dot{V}_i V_i \delta d d_i^{3-\beta}}{k_\beta} \quad (22)$$

The integration is performed from the time of injection to $t_i = \frac{k_\beta}{d_i^{3-\beta}}$ the time at which the drops vanish.

If the distribution of the injected spray follows the Rosin-Rammler expression

$$R = e^{-X^n}$$

then the volume fraction distribution

$$\text{is } v_X = -n X^{n-1} e^{-X^n}$$

$$\text{where } X = d/d_0$$

and the volume of propellant in the combustion chamber from droplet sizes between X and $X + dX$

$$\text{is } \frac{3-\beta}{6-\beta} \frac{\dot{V} d_0^{3-\beta}}{k_\beta} X^{3-\beta} n X^{n-1} dX$$

and the total volume from the whole range of spray sizes

$$\frac{3-\beta}{6-\beta} \frac{\dot{V} d_0^{3-\beta}}{k_\beta} \left(1 + \frac{3-\beta}{n}\right)$$

(23)

The volume fraction distribution function of the propellant in the chamber is

$$v'_x = \frac{n X^{n-\beta+2} e^{-X^n}}{\Gamma(1 + \frac{3-\beta}{n})} \quad (24)$$

In the two limiting cases of propellant combustion $\beta = 1$ and 2 then the volume of propellant in the combustion chamber will be given by

$$V_1 = \frac{2}{5} \cdot \frac{\dot{V} d_0^2}{k_1} \left| \Gamma(1 + \frac{1}{n}) \right| \quad (25)^*$$

$$\text{and } V_2 = \frac{1}{4} \frac{\dot{V} d_0}{k_2} \left| \Gamma(1 + \frac{1}{n}) \right| \quad (26)$$

Neither of these gamma functions is very sensitive to changes of n above the value two.

A quantity which is important when the time which the propellant spray has for combustion is limited is the volume fraction of the propellant injected at $t = 0$ remaining at time t .

From equation 21 this is given by

$$v_t = \int_x^\infty \left[1 + \left(\frac{x}{X} \right)^{3-\beta} \right]^{\frac{3-\beta}{3}} (-n) X^{n-1} e^{-X^n} dX \quad (27)$$

putting

$$\frac{d_i}{d_0} = X \quad \text{and} \quad \frac{(k_\beta t)^{\frac{1}{3-\beta}}}{d_0} = x \quad (28)$$

this equation for v_t is a function of $(k_\beta t)^{\frac{1}{3-\beta}} / d_0$ and of n and β

It can be evaluated by numerical integration.

Footnote.

This formula differs in the value of the numerical constant from that given by Probert (loc.cit). His derivation differs in the diameter variable chosen and in the paths of integration; the integration becomes cumbersome and one of the steps in the derivation is fallacious. In the notation of Probert's paper the variables are t and d the diameter at time t . Drops of diameter d may be considered to result from larger drops, injected at a time t secs. earlier, of diameter at injection $(d^2 + \lambda t)^{\frac{1}{2}}$. Then the total volume of drops of size $d \rightarrow d + \delta d$ in the steady state is

$$v_d \delta d = d^3 \int_{t=0}^{\infty} -n \frac{(d^2 + \lambda t)^{\frac{n-4}{2}}}{d_0^n} \cdot e^{-\left(\frac{d^2 + \lambda t}{d^2}\right)^{n/2}} \cdot \delta(d^2 + \lambda t)^{\frac{1}{2}} \dot{V} dt$$

Probert puts δd in place of $\delta(d^2 + \lambda t)^{\frac{1}{2}}$, confusing the alternative variables d and $d_1 = (d^2 + \lambda t)^{\frac{1}{2}}$. This expression can be integrated over values of d from zero to infinity to give the same result as (22) but the evaluation of the double integral is somewhat involved.

/APPENDIX B.

APPENDIX B.

Calculation of the rate of combustion of mono-propellant droplets.

Strictly, to evaluate the rate of combustion, we need the condition for which a solution of the partial differential equations of heat flow and mass flow exists under the appropriate boundary conditions. In the steady state, these equations may be formulated as :-

$$\text{Heat flow balance; } \text{div} (\lambda \text{ grad } T) - \text{div} (MH) + W_A Q_A = 0 \quad (1)$$

λ being the thermal conductivity

T " the temperature

M " the mass flow per unit time

H " the heat content per unit mass

W_A " the rate of reaction per unit volume of reactant A etc.

Q_A " the heat of reaction per unit mass of reactant A etc..

$$\text{Mass flow balance: } \text{div} (D_A \text{ grad } a) - \text{div} (Ma) + W_A = 0 \quad (2)$$

D_A being the diffusion constant of reactant A, a being the mass fraction of A,

$$\sum a = 1$$

For multi-component reactions, the mass flow equation can be repeated for each component.

$$\text{Finally conservation of mass: } \text{div} (M) = 0 \quad (3)$$

$$\text{and } w_A = f(a, \dots p, T), \text{ etc.} \quad (4)$$

the form of the function depending upon the kinetics of the reaction; and the appropriate boundary conditions for T and a etc., and their derivatives. Given these, and if such a stationary state is possible, a relation exists between the rate of burning and the other parameters. It has been possible to express this as an algebraic function only for certain cases, few in number, and then only with the help of drastic assumptions. For a review of these, reference should be made to Corner (39) and Markstein and Polanyi (40). A term for the flow of heat by radiation might be included in equation (1). In most systems the heat transferred by radiation is small compared with that due to conduction and the term is usually omitted. In combustion systems where the velocities of flow are low and the pressure variations throughout the system negligible, the translational kinetic energy can be neglected and the equation for the conservation of momentum omitted. Boys & Corner (11) have calculated the rate of burning of cordite by numerical integration of the equations, but only for the case of a plane surface of propellant;

/for

for the problem now considered the spherical symmetry renders the numerical integration much more laborious because the independent variable appears explicitly in the differential equations.

The difficulty in obtaining algebraic solutions lies in the exponential form of the equation connecting reaction rate and temperature which, for the case of an unimolecular reaction is :

$$w = a\rho B e^{-E/RT} \quad (5)$$

Solutions using this kinetic form have been obtained for a planar unbounded reaction zone by Frank-Kamenetski (41) but the methods used do not appear to be convenient for spherical reaction zones. We shall, therefore, employ a method of solution which is historically the oldest, being fundamentally identical with that used by Mallard and le Chatelier (1883) and with variations by other workers up to the last war. As has been pointed out by Corner (loc.cit) the approximation determines the variation of rate of reaction throughout the reaction zone and solves the heat flow equation only in the heating zone where the rate is assumed to be zero. However, if we are content with obtaining the rate of combustion per unit surface of a drop of diameter d in terms of the rate of combustion of a plane surface, the reaction rate can be eliminated from the result. The parameters necessary are a temperature T_i below which the rate of reaction is negligible and T_0 the temperature of the liquid surface. These and other assumptions are discussed on p. 10, 11 & 12. If the influence of diffusion of reactant is assumed to be negligible equation (2) need not be considered.

For a spherical drop of diameter d equation (1) becomes

$$\frac{1}{4\pi r^2} \frac{d(4\pi r^2 \lambda \frac{dT}{dr})}{dt} - \frac{\dot{M}}{4\pi r^2} \frac{d(C_p T)}{dt} + wQ \quad (6)$$

T & w being the temperature and reaction rate at a distance r from the drop centre.

The boundary conditions at the liquid surface are

$$\pi d^2 \lambda \left(\frac{dT}{dr} \right)_s = \dot{M} L, \quad T = T_0 \quad (7)$$

where L is the heat of vaporisation in calories/gram, and C_p the constant pressure specific heat in calories/degree/gram; assumed to be the same for both reactant and products.

In terms of the dimensionless variables $\theta = \frac{C_p(T - T_0 + \frac{L}{C_p})}{L}$

$$R = \frac{d}{2r}$$

$$\text{With } \xi = \frac{\dot{M} C_p}{2\pi \lambda d} \quad (8)$$

and where C_p and λ are given mean values for the temperature range covered, equation (6) may be written:

$$\frac{d^2 \theta}{dR^2} + \xi \frac{d\theta}{dR} + \frac{Q C_p d^2}{4 \lambda L} \frac{w}{R^2} = 0 \quad (9)$$

in the region $T_0 < T < T_i$, $w=0$

$$\text{and } \frac{d^2\Theta}{dR^2} + \xi \frac{d\Theta}{dR} = 0 \quad (10)$$

with boundary conditions

$$R_0=1, \Theta_0=1, \left(\frac{d\Theta}{dR}\right)_0 = \xi \quad (11)$$

the solution of (10) is

$$\Theta = e^{\xi(1-R)} \quad (12)$$

In the reaction zone we may assume $\frac{d\Theta}{dR}$ to be constant, then equation (9) becomes

$$\xi \frac{d\Theta}{dR} = \frac{QCpd^2}{4\lambda L} \cdot \frac{w}{R^4} \quad (13)$$

To eliminate $\frac{d\Theta}{dR}$ we invoke the condition that Θ must be a continuous function of R at the junction of the heating and reaction zones, then at the junction, $R = R_1$, and from (12) and (13)

$$\left(\frac{d\Theta}{dR}\right)_i = \xi \Theta = \frac{1}{\xi} \cdot \frac{QCpd^2 w_i}{4\lambda L R_i} \quad (14)$$

but from (12) $R_1 = 1 - \frac{\ln \Theta_i}{\xi}$

$$\text{whence (14) becomes } \xi^2 \left(1 - \frac{\ln \Theta_i}{\xi}\right)^4 = \frac{QCpd^2 w_i}{4\lambda L \Theta_i} \quad (15)$$

If we assume that w_i, Θ_i are independent of the radius of the drop, equation 14 is a relation between the mass rate of burning, the drop size and the other parameters of the system. Since $\dot{M} = \pi d^2 \dot{m}$ where \dot{m} is the rate of burning per unit surface, we obtain from (15)

$$\frac{\dot{m}^2 C_p}{\lambda} \left(1 - \frac{2\lambda \ln \Theta_i}{\dot{m} C_p d}\right)^4 = \frac{Q w_i}{L \Theta_i} \quad (16)$$

as d tends to infinity \dot{m} tends to \dot{m}_0 the rate of burning per unit surface for a plane surface, we may write

$$\dot{m}^2 \left(1 - \frac{2\lambda \ln \Theta_i}{\dot{m} C_p d}\right)^4 = \dot{m}_0^2 \quad (17)$$

This equation for \dot{m} has four distinct roots, given by :

$$2\dot{m} = \pm \dot{m}_0 + 2 \frac{2\lambda \ln \Theta_i}{C_p d} \pm \sqrt{\left(\pm \dot{m}_0 + 2 \frac{2\lambda \ln \Theta_i}{C_p d}\right)^2 - 4 \left(\frac{2\lambda \ln \Theta_i}{C_p d}\right)^2} \quad (18)$$

taking positive values, we see that if $4 \frac{2\lambda \ln \Theta_i}{\dot{m}_0 C_p d} \ll 1$

$$\dot{m} \approx \dot{m}_0 + \frac{2\lambda \ln \Theta_i}{C_p d} \approx \dot{m}_0 \quad (19)$$

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and if

$$4 \frac{2\lambda \ln \theta_i}{\dot{m}_o c_{pd}} \gg 1$$

$$\dot{m} \approx \frac{2\lambda \ln \theta_i}{c_{pd}} + \frac{\dot{m}_o}{2} + \left(\frac{2\lambda \ln \theta_i}{c_p} \right)^{1/2}$$

(20)

$$\approx 2\lambda \ln \theta_i / c_{pd}$$

In view of the assumptions needed for the derivation of equation (18) it cannot be expected to give more than the order of the variation of \dot{m} with diameter. It is intended to check this calculation by numerical integration of equations (1) and (2) when sufficient data is available to make the task profitable.

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